Andrei Nikolaevich Kolmogorov
April 25, 1903 - October 20, 1987
• VIDA Y OBRA
• TEORÍA DE INFORMACIÓN
K

- VIDA Y OBRA
K

- VIDA
K

Native House
K

Relatives & Aunt

• Classical Education:
  French, History
Desde 1922 enseñaba matemáticas en una escuela secundaria de Moscú y paralelamente estudiaba en la universidad. En 1922 comenzé mi investigación independiente bajo la dirección de V.V. Stepanov y N.N. Luzin. Tenía alrededor de quince publicaciones en revistas sobre teoría de funciones de variables reales al graduarme...
K

Moscow State University
K

Kolmogorov-Alexandrov

1920s

- Fourier Analysis
- Theory of Functions
- Topology
Komarovka

• Now-legendary Place: Sports, Rests
K

Middle Age

1930s

• Foundations of the Theory of Probability
• Logic
• Member of Academy at age 33
K

Wife

- She was his high-school friend
1940s

- Defense Works: Statistical Theory of Bombing & Shooting
- Second World War
- Politics
K

Wrote

1950s

- Dynamic Systems
- 13th Hilbert Problem
- Statistics
- Theory of Algorithms
1960s

- Textbooks
- Turbulence
- Linguistic Analysis of Poetry & Fiction
- Theory of Information
K

1970s

• ICM Congress in Nice
  Discrete versus
  Continuos Math
• Numerous Awards
• Main Soviet
  Mathematician
K

Smiles

• Scientific Childhood: Kolmogorov’s Enigma
Rest

- Skiing, Swimming, Sailing, Walking, Oaring...
- Classical Music (TOPAZ)
Lecture 1

- Lecturer World-Wide
Lecture 2

• Difficult to understand
Grown up several outstanding men in Math, Probability, Statistics, Logic, Informatics, Physics, Oceanography, Atmosphere, etc.
K

Kolmogorov Students

All Soviet Union
Kolmogorov’s High School in Physics & Math # 18
Founded on December 2, 1963
Awards

- Seven Orders of Lenin, the Order of the October Revolution, and also the high title of Hero of Socialist Labor; Lenin Prizes and State Prizes.
Awards

- The Royal Netherlands Academy of Sciences (1963), the London Royal Society (1964), the USA National Society (1967), the Paris Academy of Sciences (1968), the Polish Academy of Sciences, the Rumanian Academy of Sciences (1956), the German Academy of Sciences Leopoldina (1959), the American Academy of Sciences and Arts in Boston (1959).
Awards

- Honorary doctorates from the universities of Paris, Berlin, Warsaw, Stockholm, etc.
- Honorary member of the Moscow, London, Indian, and Calcutta Mathematical Societies, of the London Royal Statistical Society, the International Statistical Institute, and the American Meteorological Society.

Ya soñaron un segundo... y ¡mano a la obra!
1980s

• Health Problems (Parkinson)
Elder Age

Died in Moscow on October 20, 1987†
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• OBRA
• Algebra and Elements of Analysis (A High School Textbook for 10-11 grades) Edited by A. N. Kolmogorov et al Published by Prosveshchenie, 2001 (11th Edition)
K

- Quantum: The Magazine of Math and Science A. N. Kolmogorov co-founded in 1970 and was First Deputy Editor-in-Chief of the Russian popular scientific journal for students 'Kvant'.
Elements of the Theory of Functions and Functional Analysis by A. N. Kolmogorov and S. V. Fomin Published March 1999 by Dover
• Theory of Probability and Its Applications A. N. Kolmogorov founded in 1956 and was Editor-in-Chief of the Russian journal 'Teoriya Veroyatnostei i ee Primeneniya'. Current Editor-in-Chief is Yu. V. Prokhorov who is a Kolmogorov's student.
• Russian Mathematical Surveys A. N. Kolmogorov was a founding member of the editorial board of the Russian journal 'Uspekhi Matematicheskikh Nauk' from 1934 till his death in 1987. He was Editor-in-Chief from 1946 till 1954 and from 1982 to 1987.
K

- Kolmogorov in Perspective Edited by A. N. Shiryaev
  Published September 2000 by American Mathematical Society
K

- KOLMOGOROV
  Volume 1 edited by A. N. Shiryaev
  published in Moscow, May 2003
K

• **KOLMOGOROV**
  Volume 2 edited by A. N. Shiryaev
  published in Moscow, May 2003
K

- **KOLMOGOROV**
  Volume 3 edited by A. N. Shiryaev
  published in Moscow, May 2003
• An Extraordinary Event - A book about A. N. Kolmogorov edited by V. M. Tikhomirov, published in Moscow 1999
• V. A. Uspensky
  Works on Non-Mathematics (with appendix: Semiotic epistles of A. N. Kolmogorov to the author and his friends) Published by OGI, Moscow, 2002
K

• TURBULENCE: The Legacy of A. N. Kolmogorov by Uriel Frisch Published 1994 by Cambridge University Press
K

• L'héritage de Kolmogorov en physique Edited by Roberto Livi and Angelo Vulpiani
  Published September 2003 by Belin, Paris
In memoriam... K-100

• Scientist's Club of the Moscow State University and Moscow Mathematical Society Kolmogorov Centennial Memorial Meeting Great Hall of the Moscow State University (Moscow, April 29, 2003)

• Russian Academy of Sciences (RAS) and Moscow State University International Conference Kolmogorov & Contemporary mathematics (Moscow, June 16 - 21, 2003)
In memoriam... K-100

• International conference devoted to the hundredth anniversary of A.N. Kolmogorov's birth (Tambov, May 11-16, 2003) General Problems of Optimal Control and Their Applications

• Third Annual Moscow Kolmogorov Readings - 2003 International Science Students Conference Kolmogorov Specialized Physics and Mathematics School - Internat #18, Moscow State University May 5-7, 2003, Moscow
In memoriam... K-100

- Kolmogorov's Legacy in Physics: One Century of Chaos, Turbulence and Complexity 15-17 September 2003, ICTP, Trieste, Italy
- One-Day Workshop in Honor of the One Hundredth Anniversary of the Birth of Andrei Nikolaevich Kolmogorov Complexity, Information, and Randomness: The Legacy of Andrei Kolmogorov Sunday, July 6, 2003, University of Aarhus, Denmark in conjunction with 18th Annual IEEE Conference on Computational Complexity Monday, July 7th, to Thursday, July 10th, 2003, University of Aarhus, Denmark
In memoriam... K-100

- **Centennial Seminar on Kolmogorov Complexity and Applications** International Conference and Research Center for Computer Science 2003, April 27 - May 5, Wadern, Saarbrücken, Germany

- Il Centro per la Meccanica Statistica e la Complessita' (SMC) dell' Istituto Nazionale per la Fisica della Materia (INFM) e il Dipartimento di Fisica Universit'a degli Studi di Roma La Sapienza organizzano una Giornata su **L'EREDITA' CULTURALE di A.N. KOLMOGOROV** La Conferenza si terr'a presso l'Edificio Fermi del Dip di Fisica, Roma AULA 1 ore 16 del 9 Maggio 2003
In memoriam... K-100

- The University of London Inaugural Kolmogorov Lecture Speaker: Professor Ray Solomonoff 27th February 2003 5:30pm, Main Lecture Theatre Royal Holloway, University of London
- Fields Institute Kolmogorov Lecture Series 1998-1999
- Chicago Kolmogorov Memorial Readings 2001
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• TEORÍA DE INFORMACIÓN
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• OBRAS SELECTAS
- Selected Works of A.N.Kolmogorov: Mathematics and Mechanics, Volume 1 Edited by V. M. Tikhomirov
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- INFORMACIÓN₀

Turing (TM)
Shannon (log₂N)
Kolmogorov (LA)

Preliminary K-definition & Reflections on it (Looks & Smiles)
2^{100} messages by Shannon contain \( \log_2 2^{100} = 100 \) bits of info (\( \log_2 N \))!?
• **MOTIVACIÓN**

Shannon’s entropy
(“amount of information”)

\[ - \sum_{i, p_i \neq 0} p_i \log_2(p_i) \]
Where $p_i$ is the probability of occurrences of value $i$. Using this criterion, the higher the entropy, the more the randomness. For instance, the sequence

```
00100010
```

(entropy $= 0.81$) is less random than

```
01010101
```

(entropy $= 1$). The *inadequacy* of Shannon's measure of randomness is apparent, because intuitively the former sequence is more random than the latter.
K

• **K-DEFINICIÓN**

We start by describing objects with binary strings (Turing). We take the length of the shortest string description of an object as a measure of that *object's complexity*. 
Let's look at the most traditional application of information theory, communications (Shannon). The sender and receiver each know specification method $L$. Message $x$ can be transmitted as $y$, such that $L(y) = x$. This is written "$y : L(y) = x$" and the ":" is read as "such that". The cost of transmission is the length of $y$, $|y|$. The least cost is

$$\min(|y|): L(y) = x$$
This minimal |y| is the descriptional complexity of x under specification method L. A universal description method should have the following properties:

- it should be independent of L, within a constant offset, so that we can compare the complexity of any object with any other object's complexity;

- the description should in principle be performable by either machines or humans.
Such a method would give us a measure of absolute information content, the amount of data which needs to be transmitted in the absence of any other \textit{a priori} knowledge. The description method that meets these criteria is the \textit{Kolmogorov complexity}: the size of the shortest program (in bits) that without additional data, computes the string and terminates.

\[ K(x) = \min |p| : U(p) = x \]

where \( U \) is a universal Turing machine (believe!).
The conditional Kolmogorov complexity of string \( x \) given string \( y \) is

\[
K(x|y) = \min_{p} |p| : U(p,y) = x
\]

The length of the shortest program that can compute both \( x \) and \( y \) and a way to tell them apart is

\[
K(x,y) = \min_{p} |p| : U(p) = xy
\]
The Kolmogorov information of string $x$ given string $y$ is

$$I (x|y) = K(y) - K(y|x)$$
\[ K \]

**EXAMPLE (COM)_1**

\( \pi \) is an infinite sequence of seemingly random digits, but it contains only a few bits of information: the size of the short program that can produce the consecutive bits of \( \pi \) forever. Informally we say the descriptional complexity of \( \pi \) is a constant. Formally we say \( K(\pi) = 0(1) \), which means "\( K(\pi) \) does not grow".
\textbf{EXAMPLE (COM)$_2$}

A truly random string is not significantly compressible; its description length is within a constant offset of its length. Formally we say $K(x) = \Theta(|x|)$, which means "$K(x)$ grows as fast as the length of $x$"
The amount of information contained in the variable $x$ with respect to a related variable $y$ can be defined as follows.
The relationship between variables $x$ and $y$ ranging over the respective sets $X$ and $Y$ is that not all pairs $x,y$ belonging to the Cartesian product $X \times Y$ are “possible”. From the set of possible pairs $U$, one can determine, for any $a \in X$, the set $Y_a$ of those $y$ for which $(a,y) \in U$.

It is natural to define the conditional entropy by the relation

$$H(y|a) = \log_2 N(Y_a)$$

where $N(Y_a)$ is the number of elements in the set $Y_x$. 
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$$H(y|a) = \log_2 N(Y_a)$$

where $N(Y_x)$ is the number of elements in the set $Y_x$ and the information in $x$ with respect to $y$ by the formula

$$I(x|y) = H(y) - H(y|x)$$
It is natural to define the conditional entropy by the relation

$$H(y|a) = \log_2 N(Y_a)$$

where $N(Y_x)$ is the number of elements in the set $Y_x$ and the information in $x$ with respect to $y$ by the formula

$$I(x|y) = H(y) - H(y|x)$$
### Example (INFO)_{35}

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<thead>
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<td>2</td>
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<td>3</td>
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For example, in the case shown in the table, we have

\[ I(x = 1|y) = 0, \ I(x=2|y) = 1, \]
\[ I(x = 3|y) = 2. \]

It is clear that \( H(y|x) \) and \( I(x|y) \) are functions of \( x \) (while \( y \) enters in them as a "bound variable").
K

- **CONCLUSIÓN**

  Combinatorial (C)
  Probabilistic (P)
  Algorithmic (A)

K-theorem:
C → A;  P → A
Algorithmic Probability &
Induction
Prediction
Occam’s Razor
Distance
Superficiality & Sophistication
Part & Whole
Total & Partial Order
Computable description methods
GOOD BYE A BEAUTIFUL MIND!